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CS534-S23-S01 Group Project Assignment #3

**Question 1. *Convert the following set of sentences into the conjunctive normal form (CNF).***

S1: A  (B ⋁ E)

Biconditional elimination: (A  B ⋁ E) ⋀ (B ⋁ E  A)

Implication elimination: (A ⋁ B ⋁ E) ⋀ ( (B ⋁ E) ⋁ A)

De Morgan: (A ⋁ B ⋁ E) ⋀ (B ⋀ E) ⋁ A)

Distributivity Law: (A ⋁ B ⋁ E) ⋀ (B ⋁ A) ⋀ (E ⋁ A)

CNF: (A ⋁ B ⋁ E) ⋀ (B ⋁ A) ⋀ (E ⋁ A)

S2: E  D

Implication elimination: E ⋁ D

CNF: E ⋁ D

S3: C ⋀ F  B

Implication elimination: (C ⋀ F) ⋁ B

De Morgan: (C ⋁ F) ⋁ B

CNF: C ⋁ F ⋁ B

**Question 2. *Assuming predicates Parent(p, q) and Female(p) and constants Joan and Kevin, with the obvious meanings, express each of the following sentences in first-order logic. (You may use the abbreviation ∃1 to mean “there exists exactly one.”)***

1. *Joan has a daughter (possibly more than one, and possibly sons as well).*

∃x Female(x) ⋀ Parent(Joan,x)

There exists an x such that x is a female and Joan is the parent of x.

1. *Joan has exactly one daughter (but may have sons as well).*

∃1x Female(x) ⋀ Parent(Joan,x)

There exists exactly one x such that x is a female and Joan is the parent of x.

1. *Joan has exactly one child, a daughter.*

∃1x Parent(Joan,x) ⋀ Female(x)

There exists exactly one x such that Joan is the parent of x and x is a female.

1. *Joan and Kevin have exactly one child together.*

∃1x Parent(Joan,x) ⋀ Parent(Kevin,x)

There exists exactly one x such that Joan is the parent of x and Kevin is the parent of x.

1. *Joan has at least one child with Kevin, and no children with anyone else.*

(∃x Parent(Joan,x) ⋀ Parent(Kevin,x)) ⋀ ­­­­­­­­­­­(∃y Parent(Joan,y) ⋀ Parent(Kevin,y))

There exists an x such that Joan is the parent of x and Kevin is the parent of x and there doesn’t exist a y such that Joan is the parent of y and Kevin is not the parent of y.

**Question 3. *Five Customers’ Rating on a New Car on a 20-point Scale.***

Chart, box and whisker chart

Description automatically generated

1. *Assume K = 2 and the two initial centroids are 3 and 4.*



1. Clusters = {2,5} and {9,10,15}
2. Silhouette Coefficient



Davies-Bouldin Index



Calinski-Harabasz Index



1. *Assume K = 2 and the two initial centroids are 11 and 12.*



1. Clusters = {2,5,9,10} and {15}
2. Silhouette Coefficient



Davies-Bouldin Index



Calinski-Harabasz Index



1. *Use the results from (a) and (b) to determine which two-cluster solution should be chosen. Please describe and explain your answer in detail.*

Comparing the Silhouette Coefficient Index, Davies–Bouldin Index, and Calinski-Harabasz Index of each set of clusters, it would appear that {2,5} and {9,10,15} should be chosen. This cluster solution offers both a higher Silhouette and Calinski-Harabasz Index. The Davies-Bouldin Index is skewed toward for {2,5,9,10} and {15} since the average distance to the centroid for the {15} cluster is 0 lowering the DB Index below that of {2,5} and {9,10,15}. The {2,5} and {9,10,15} option also offers a lower SSE value of 25.16 compared to 41 for {2,5,9,10} and {15}.